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Resonances and intensity-dependent shifts of the Møller cross section in a strong laser field

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Abstract. The laser field is treated as a classical external field and represented as a monochromatic plane-wave field of circular polarisation. The differential cross section for the scattering of unpolarised electrons in this field is calculated and numerically evaluated for the non-relativistic energy region. Because of the complicated kinematics of this process an experimental situation is considered where the incoming electrons have opposite and equal momentum. It is shown that the off resonance cross section in a strong field can differ considerably from the free case in regions accessible to experimentation for various laser and electron parameters. Resonances are found to exist in physical regions. Renormalisation is performed. The resonance cross section, the width and the spacing of the resonances are evaluated. Under specific circumstances the resonance cross section is found to exceed the off-resonance cross section by several orders of magnitude. The experimental conditions for the resonance case, however, seem to be harder to meet than in the off resonance case.

1. Introduction

In the presence of an intense laser field the electromagnetic interaction processes of particles are changed. The theory of strong fields developed in the last decade predicts intensity-dependent corrections. So far, however, it has been impossible to measure these corrections, either because they were too small or because lasers of the required intensity did not exist. Due to the rapid development of high power lasers it should soon be possible to test the theory experimentally. Especially interesting in this context are the resonances predicted for Møller scattering by Oleinik (1967, 1968). His work was, however, largely theoretically orientated. A numerical value of the resonant cross section was given for only one point in an experimentally completely inaccessible kinematic region. The question remained whether resonances can be predicted for physically reasonable electron and laser parameters.

In this paper numerical results for resonances and intensity-dependent shifts of the cross section are presented which should be large enough for experimental verification. The experimental conditions for resonances, however, may be difficult to achieve.

The calculations are based on the formalism published by Mitter (1975) in a summary of quantum electrodynamics in strong laser fields. Perturbation theory is used for all non-laser quanta, but not with respect to the laser field, since the coupling constant is not small for intense fields. The laser field is represented as an external (classical) plane wave field. In order to simplify the calculations the special case with the following preconditions is treated:

—circularly polarised laser field
—unpolarised electrons
—incoming electrons with opposite and equal momenta.

For technical reasons the numerical calculations could be performed only for the non-relativistic energy region.

For intense fields the external field approximation is quite good as the number of laser photons is large. The approximation ceases to be valid when the laser beam is considerably depleted by the scattering process. The description of the laser field as a plane wave is more problematical. Difficulties arise mainly in formulating the boundary conditions since the particles are not asymptotically free. For high-intensity Compton scattering Neville and Rohrlich (1970) could show that the results of the plane wave method are essentially correct.

A monochromatic plane wave with fixed propagation direction $\boldsymbol{n}$ is characterised by the wave vector $k^\mu = \omega(1, \boldsymbol{n})$, ω being the laser frequency. The vector potential for arbitrary polarisation is given by

$$A^\mu(x) = a \sum_{i=1}^{2} e_i^\mu a_i(\xi)$$

with amplitude a, polarisation vector e_i^μ and $\xi = k^\mu x_\mu$. For the following calculations circular polarisation is chosen: $a_1(\xi) = \cos\xi$, $a_2(\xi) = -\sin\xi$. In this case the inevitably lengthy expressions for the matrix elements reduce to a certain extent due to an additional symmetry (Mitter 1975).

A frequently employed measure for the intensity of the laser field is the (classical) parameter

$$\nu^2 = (ea/mc^2)^2.$$

In technical units: $\nu^2 \approx 7{\cdot}5 \times 10^{-11} \lambda^2 I$, where the wavelength λ is in cm and the illumination density I in watt cm^{-2}. Due to recent developments in laser technique a ν^2 of magnitude one appears to be attainable.

2. The Møller amplitude

The lowest order Feynman graphs for Møller scattering are (Figure 1):

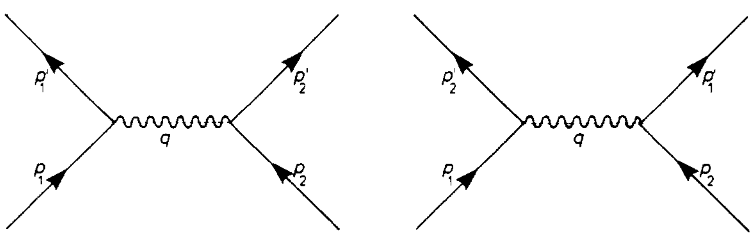

Figure 1. $d\sigma/d\Omega$ (10^{-16} cm^2) as a function of the intensity parameter ν^2 for two laser frequencies and $E_{\rm kin} = 5{\cdot}1$ eV, $\psi = 45°$, $\phi = 90°$, $\theta = 90°$. Upper curve: $\omega = 1{\cdot}9 \times 10^{15}$ s^{-1}. Lower curve: $\omega = 8 \times 10^{15}$ s^{-1}.

For the electron–electron scattering in an external field the electron lines can be replaced by the Volkov solutions (Volkov 1935). Here we shall use an equivalent formulation in momentum space instead, the E-representation (cf Mitter 1975). As above the free solutions are inserted for the electron and photon lines, but instead of the usual vertex $\gamma^\mu\delta(p'-p+q)$ we have to use the modified expression

$$v^\mu(p'p|q)=(2\pi)^{-4}\int \mathrm{d}^4x\bar{E}(x|p')\gamma^\mu E(x|p)\exp(\mathrm{i}q\,.\,x)$$

with

$$E(x|p)=\exp(-\mathrm{i}p\,.\,x)[1-(ea/2p_v)\gamma_v\gamma_ia_i(u)]$$
$$\times\exp\Big[\mathrm{i}(ea/p_v)\Big(p_i\int_{u_0}^{u}a_i(\bar{u})\,\mathrm{d}\bar{u}-\tfrac{1}{2}ea\int_{u_0}^{u}a_i^2(\bar{u})\,\mathrm{d}\bar{u}\Big)\Big]$$

for a laser field of arbitrary polarisation†.

Then the matrix element for Møller scattering in an external field is

$$S_{fi}=-\mathrm{i}e^2\frac{(2\pi)^2}{v^2}\Big(\frac{m^4}{E_1E_1'E_2E_2'}\Big)^{1/2}\int \mathrm{d}^4q\{[\bar{u}(p_1',s_1')v^\mu(p_1'p_1|q)u(p_1,s_1)]D_{\mu\nu}(q^2)$$
$$\times[\bar{u}(p_2',s_2')v^\nu(p_2'p_2|-q)u(p_2,s_2)]-[(p_1',s_1')\leftrightarrow(p_2',s_2')]\}. \qquad (2.1)$$

For circular polarisation the integrals in $E(x|p)$ are elementary. All integrals in $v^\mu(p'p|q)$ can be transformed to δ-functions, if we use the expansion

$$\exp[\mathrm{i}\lambda(\sin\xi+\rho)]=\sum_{n=-\infty}^{+\infty}\mathrm{J}_n(\lambda)\exp[\mathrm{i}n(\xi+\rho)]$$

and if in addition the laser is assumed to be infinitely extended in the u direction. This last step produces infinite phase factors which, however, drop out in $|S_{fi}|^2$, so that we shall omit them from now on. By introducing the definitions

$$G_n^\mu(p',p)=\mathrm{J}_n(\lambda(p',p))B^\mu(p',p)-\mathrm{J}_{n-1}(\lambda(p',p))\exp(-\mathrm{i}\rho(p',p))C_+^\mu(p',p)$$
$$-\mathrm{J}_{n+1}(\lambda(p',p))\exp(\mathrm{i}\rho(p',p))C_-^\mu(p',p)$$

$$\lambda(p',p)=\frac{ea}{2^{1/2}\omega}\frac{(p_v'^2p_i^2+p_i'^2p_v^2-2p_v'p_vp_i'p_i)^{1/2}}{p_v'p_v},\ \rho(p',p)=\tan^{-1}\frac{p_2'p_v-p_v'p_2}{p_1'p_v-p_v'p_1} \qquad (2.2)$$

$$B^\mu(p',p)=\gamma^\mu+\frac{(ea)^2}{2p_v'p_v}n^\mu\gamma_v, \qquad C_\pm^\mu=\frac{ea}{2^{3/2}}((p_v')^{-1}\gamma_\pm\gamma_v\gamma^\mu+(p_v)^{-1}\gamma^\mu\gamma_v\gamma_\pm)$$

and the effective momenta $\tilde{p}$ (cf Becker 1976)

$$\tilde{p}=p+\frac{(ea)^2}{2p\,.\,k}k \quad \text{with } \tilde{p}^2=m^2(1+\nu^2)=m_*^2$$

v^μ takes the final form

$$v^\mu(p'p|q)=\sum_{n=-\infty}^{\infty}\delta^4(\tilde{p}'-\tilde{p}+q+nk)\exp(\mathrm{i}n\rho(p',p))G_n^\mu(p',p).$$

Instead of the momentum conservation $p'-p+q=0$ at the vertex γ^μ without an external field, we now have $\tilde{p}'-\tilde{p}+q+nk=0$. Thus the laser affects an electron in two

† We employ natural units ($\hbar=c=1$) and a metric such that $a\,.\,b=a_\mu b^\mu=a_0b_0-\boldsymbol{a}\,.\,\boldsymbol{b}$. A definition of light-like coordinates and Dirac matrices is given in Appendix A.

ways: the mass of the electron is changed ($m \to m_*$), and the electron can emit or absorb n laser quanta. Inserting the expression for v^μ into (2.1) we obtain after integration

$$S_{fi} = \mathrm{i}e^2 \frac{2\pi^4}{v^2}\left(\frac{m^4}{E_1E_2E_1'E_2'}\right)^{1/2} \sum_{n,n'=-\infty}^{\infty} \delta^4(\tilde{p}_1' + \tilde{p}_2' - \tilde{p}_1 - \tilde{p}_2 + [n+n']k)H_{n,n'}$$

$$\begin{aligned} H_{n',n} = {} & \exp(\mathrm{i}n\rho(p_1', p_1) + \mathrm{i}n'\rho(p_2', p_2)) \\ & \times \frac{[\bar{u}(p_1', s_1')G_n^\mu(p_1', p_1)u(p_1, s_1)][\bar{u}(p_2', s_2')G_{n'\mu}(p_2', p_2)u(p_2, s_2)]}{(\tilde{p}_1' - \tilde{p}_1 + nk)^2} \\ & -[(p_1', s_1') \leftrightarrow (p_2', s_2')]. \end{aligned} \tag{2.3}$$

With the total number $r = -(n+n')$ of laser photons absorbed or emitted by both electrons, the energy–momentum conservation of the scattering process reads

$$\tilde{p}_1' + \tilde{p}_2' - \tilde{p}_1 - \tilde{p}_2 - rk = 0. \tag{2.4}$$

The effect of this—compared with the free Møller scattering—strongly modified conservation law on the energies of the scattered electrons will be examined in detail elsewhere (Bös, Brock, Mitter and Schott 1978).

Another interesting feature is that S_{fi} and hence also the cross section exhibit a resonance behaviour if one of the denominators of S_{fi} becomes zero (cf Oleinik 1967):

$$q^2 = (\tilde{p}_1' - \tilde{p}_1 + nk)^2 = 0$$

or, equivalently,

$$q^2 = (\tilde{p}_2' - \tilde{p}_2 + nk)^2 = 0. \tag{2.5}$$

As the Feynman graphs show, q is the momentum of the virtual photon exchanged between the electrons.

According to (2.5) resonance is to be expected when the emission of the photon at one vertex and its absorption at the other one can take place as a real process. The resonances are treated in more detail in § 5.

3. The cross section

The cross section is calculated in the usual way from the square of the Møller amplitude, averaged over the initial electron spin states s_1, s_2 and summed over the final spin states s_1', s_2'. Similar to simpler processes in laser fields the cross section is a sum of incoherent contributions $\mathrm{d}\sigma_r$ corresponding to the r-dependent energies of the scattered electrons:

$$\mathrm{d}\sigma = \sum_{r=-\infty}^{+\infty} \mathrm{d}\sigma_r$$

with

$$\begin{aligned} \mathrm{d}\sigma_r = {} & \frac{e^4}{2\pi^2[(p_1 \cdot p_2)^2 - m^4]^{1/2}} \int \frac{\mathrm{d}^3p_1'}{E_1'} \int \mathrm{d}^4p_2'\delta^4 \\ & \times (\tilde{p}_1' + \tilde{p}_2' - \tilde{p}_1 - \tilde{p}_2 - rk)^2\delta(p_2' - m^2)\theta(E_2')M_r, \end{aligned}$$

$$M_r = \tfrac{1}{4}m^4 \sum_{s_1,s_2,s_1',s_2'} \left| \sum_{n=-\infty}^{+\infty} H_{n,-r-n} \right|^2.$$

Due to the external field approximation the laser field can absorb or emit arbitrarily high energies, hence the infinite sum over r. Contrary to the field the electrons cannot emit an arbitrary number of photons because of their limited kinetic energy. This implies a lower limit for r (Brock 1977). A definite upper limit for r cannot be deduced. The assumption that the scattering process will not change the field will, however, only be valid for values of r which are not too large. The convergence in r will be discussed later in this section.

The integration over p_2' can easily be performed if the transformation $p_2' \to \tilde{p}_2'$ is applied; the Jacobian is 1. Using $p_2'^2 - m^2 = \tilde{p}_2'^2 - m_*^2 = 0$, $\mathrm{d}^3 p_1' = p_1'^2\, \mathrm{d}p_1'\, \mathrm{d}\Omega$ and $|\boldsymbol{p}_1'|\, \mathrm{d}p_1' = E_1'\, \mathrm{d}E_1'$, we obtain

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}\Omega} = \frac{e^4}{2\pi^2((p_1 \cdot p_2)^2 - m^4)^{1/2}} \int \mathrm{d}E_1' |\boldsymbol{p}_1'|\, \delta[(\tilde{p}_1 + \tilde{p}_2 - \tilde{p}_1' + rk)^2 - m_*^2] M_r. \tag{3.1}$$

The last integration requires a solution E_1' of the equation

$$(\tilde{p}_1 + \tilde{p}_2 - \tilde{p}_1' + rk)^2 - m_*^2 = 0. \tag{3.2}$$

Without the external field the argument of the δ-function is $[(p_1 + p_2 - p_1')^2 - m^2]$ which leads to a quadratic equation in E_1'. In the centre of mass system (CMS) the integration is trivial because $E_1 = E_2 = E_1' = E_2'$. Due to the form of the effective momenta $\tilde{p}$ the energy condition (3.2) is now much more complicated and includes values of E_1' to the fourth power. The introduction of a CMS for the incoming electrons by the definitions $p_1 = (E, \boldsymbol{p})$, $p_2 = (E, -\boldsymbol{p})$ does not reduce these difficulties, as in general neither $E = E_1'$ nor $E = E_2'$ or $E_1' = E_2'$ will be valid. Therefore Oleinik used an auxiliary system by redefining $\tilde{p}_2' := p_2' + (ea)^2/2p_2' \cdot k - rk$ and setting $\tilde{\boldsymbol{p}}_1 + \tilde{\boldsymbol{p}}_2 = \tilde{\boldsymbol{p}}_1' + \tilde{\boldsymbol{p}}_2' = 0$. Then the last integration is easily carried out, but the transformation back to a physical reference system is difficult. Even in the non-relativistic limit the relations between the angles in the auxiliary system and the laboratory frame are very complicated.

Here a physical reference system shall be used from the start, the CMS of the incoming electrons (in which all formulas are shorter than in the laboratory system). In the limit of a vanishing external field this CMS goes over to the normal CMS. The momenta are

$$p_1 := (E, \boldsymbol{p}), \qquad p_2 := (E, -\boldsymbol{p}), \qquad p_1' = (E_1', \boldsymbol{p}_1'), \qquad p_2' = (E_2', \boldsymbol{p}_2')$$

and the angles are defined by

$$\psi = \measuredangle(\boldsymbol{k}, \boldsymbol{p}), \qquad \phi = \measuredangle(\boldsymbol{k}, \boldsymbol{p}_1'), \qquad \theta = \measuredangle(\boldsymbol{p}, \boldsymbol{p}_1').$$

The scattering angle θ must satisfy the condition

$$|\psi - \phi| \leq \theta \leq \min(\psi + \phi, 2\pi - \psi - \phi).$$

The definition describes an experimental situation where the incoming electrons are shot against one another with equal and opposite momenta and interact within the laser field. For the calculation of the cross section E, ψ, φ and θ must be given. E_1' and E_2' have to be determined numerically, since the energy condition (3.2) can only be solved without approximations in very specific cases. If a solution E_1' is known, the last integration in (3.1) can be performed and we obtain

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}\Omega} = \frac{e^4}{(2\pi)^2} \frac{(E_1'^2 - m^2)^{1/2}}{E(E^2 - m^2)^{1/2}} \left| \frac{\mathrm{d}\tilde{p}_2'^2}{\mathrm{d}E_1'} \right|^{-1} M_r. \tag{3.3}$$

The sum over the electrons spins may be used as usual to convert M_r into traces over Dirac matrices. Here this procedure has several disadvantages. One arrives at traces with products of up to sixteen matrices so that the general result is far too long to permit an analysis of the structure of the cross section or to serve as a starting point for numerical calculations. Even worse is an additional infinite sum.

Inserting (2.3) and the definitions (2.2) into M_r the matrix parts and the terms depending on the index n can be dealt with separately. The numerical treatment of the matrix parts is long but not difficult. All n-dependent terms have a form similar to

$$\sum_{n=-\infty}^{+\infty} \frac{J_n(\lambda_1)J_{-r-n}(\lambda_2)\exp(-\mathrm{i}nx)}{(\tilde{p}_1' - \tilde{p}_1 + nk)2}$$

with $\lambda_i := \lambda(p_i', p_i)$ (i = 1, 2) and $x = \rho(p_2', p_2) - \rho(p_1', p_1)$. For $p_1' . k \neq p_1 . k$ the resonance denominator can be written

$$(\tilde{p}_1' - \tilde{p}_1 + nk)^2 = 2(p_1 . k - p_1' . k)(b - n),$$

$$b = \frac{p_1' . p_1 + \frac{1}{2}(ea)^2\left(\dfrac{p_1' . k}{p_1 . k} + \dfrac{p_1 . k}{p_1' . k}\right) - m_*^2}{p_1' . k - p_1 - k}.$$

Thus the n-dependent terms reduce to

$$S_r(b) = \sum_{n=-\infty}^{+\infty} \frac{J_n(\lambda_1)J_{-r-n}(\lambda_2)\exp(-\mathrm{i}nx)}{b-n}.$$

An explicit expression for M_r is given in Appendix B. In the special case $p_1' . k = p_1 . k$ one has $(\tilde{p}_1' - \tilde{p}_1 + nk)^2 = 2(m^2 - p_1' . p_1)$, independent of n. Then $S_r(b)/(p_1 . k - p_1' . k)$ in M_r must be replaced by $T_r/(p_1' . p_1 - m^2)$ with

$$\begin{aligned} T_r &= \sum_{n=-\infty}^{+\infty} J_n(\lambda_1)J_{-r-n}(\lambda_2)\exp(-\mathrm{i}nx) \\ &= \left(\frac{\lambda_2 + \lambda_1 \exp(-\mathrm{i}x)}{\lambda_2 + \lambda_1 \exp(\mathrm{i}x)}\right)^{-r/2} J_{-r}(\lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2 \cos x)^{1/2} \end{aligned}$$

(cf Watson 1966). There are no divergences in this case apart from the Coulomb divergence ($\theta = 0°$, $\phi = \psi$, $E = E_1'$) which also exists in free Møller scattering. Unfortunately no closed form could be found for $S_r(b)$. Approximations are hard to obtain because of the large range of values of b and λ. Therefore the summation had to be done directly by the computer. For the convergence of the series $S_r(b)$ the properties of the Bessel functions are decisive. For fixed λ the magnitude of $J_n(\lambda)$ remains roughly the same between $n = 0$ and $|n| \approx \lambda$, but converges rapidly to zero for $|n| > \lambda$. In order to reach the convergence region, many terms have to be added because the argument λ (§ (3.2)) becomes very large for high laser intensity and high electron energies. For a Nd–glass laser, e.g. with $\omega = 1{\cdot}9 \times 10^{15}\ \mathrm{s}^{-1}$, $\nu^2 = 0{\cdot}1$ and $r = 0$, we find $\lambda \approx 10^3$ for a kinetic energy of 5 eV, but $\lambda \approx 10^6$ for 5 MeV. As λ is inversely proportional to ω, λ is smaller for higher frequencies. Therefore a part of the calculations was made for $\omega = 8 \times 10^{15}\ \mathrm{s}^{-1}$, although high power ultraviolet lasers probably do not yet exist. But even then the summation of so many terms is extremely uneconomical and not very accurate. For these reasons the calculations of § 4 and 5 had to be restricted for high intensities to non-relativistic energies (500 eV at most, i.e. $n \lesssim 30\,000$). The relativistic formulation of $d\sigma/d\Omega$ is too clumsy for this energy region and non-relativistic expansions are needed for the numerics (Brock 1977).

4. Results for the off resonance cross section

As mentioned at the beginning of § 3 the cross section is an incoherent sum

$$\frac{d\sigma}{d\Omega}=\sum_r \frac{d\sigma_r}{d\Omega}.$$

Similar to the n-convergence of $S_r(b)$, the convergence of this sum is determined by the properties of the Bessel functions and depends on the electron energies and the laser intensity. In principle the single contribution $d\sigma_r/d\Omega$ can be measured separately since the energies of the scattered electrons are different for each r. This would require, however, a very high resolution since the energy difference for two consecutive values of r is of the magnitude of the energy of a laser photon. Therefore only approximation values of the sum $d\sigma/d\Omega$ are presented in this section.

For figures 2 and 3, ψ and ϕ were chosen in such a way that the difference between $d\sigma/d\Omega$ and the free cross section is as large as possible.

Figure 2 shows that ν^2 dependence of $d\sigma/d\Omega$ for two laser frequencies. Low energy electrons hit the laser beam at an angle of 45° to the laser axis. They are detected in a plane perpendicular to the laser axis and at a scattering angle of 90°. For both frequencies the cross section may be changed by 100%, if the laser field is sufficiently strong. The deviations from the free case ($\nu^2 = 0$) turn out to be even larger for a kinetic energy of 25·5 eV so that this intensity-dependent effect is not restricted to very low energies of the electrons and should be accessible for experimental verification. Because of technical difficulties no predictions could be made for the middle or high relativistic energy region.

In figure 3 the dependence of the cross section on the scattering angle θ is presented. Again the electrons are shot into the laser beam at an angle of 45° and detected in a

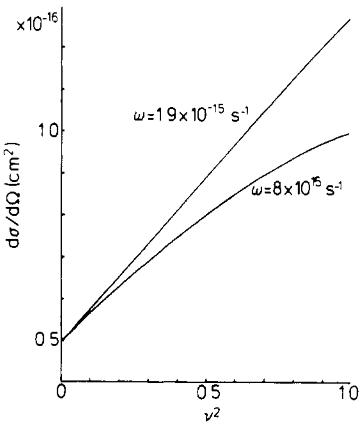

Figure 2. $d\sigma/d\Omega$ (10^{-16} cm^2) and the cross section without laser field, $(d\sigma/d\Omega)$ free, as a function of the scattering angle θ for $\omega = 8.0\times10^{15}$ s^{-1}, $\nu^2 = 1$, $E_{kin} = 5\cdot1$ eV, $\psi = 45°$, $\phi = 90°$.

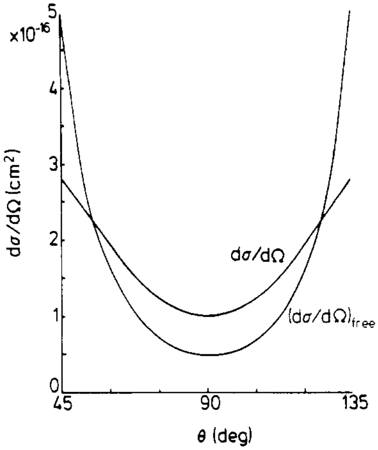

Figure 3. ν^2 dependence of the resonance cross section (in units cm^2) for $\omega = 1{\cdot}9\times10^{15}\ \mathrm{s}^{-1}$, $E_{\mathrm{kin}} = 5.1$ eV, $\psi = 45°$, $\phi = 44{\cdot}83°$, $\theta = 0{\cdot}17°$.

plane perpendicular to the direction of the incoming electrons. The diagram shows that in figure 2 a suitable angle, $\theta = 90°$, was chosen. For higher energies the graphs look similar. For measurements it is important to notice that $\mathrm{d}\sigma/\mathrm{d}\Omega$ varies near $\theta = 90°$ only slightly with small changes in θ; the same applies for the corresponding values of ψ and ϕ.

5. Resonances

In § 2 the resonance conditions (2.5) were derived. Before starting to calculate the renormalised cross section at a resonance point, the question must be examined whether resonances may exist at all for physically reasonable parameters for the electrons and the laser field.

Inserting the expressions for the effective momenta into the first resonance condition and resolving for cos θ, we obtain the equation

$$\cos\theta = [(E^2-m^2)(E_1'^2-m^2)]^{-1/2}$$
$$\times\left\{EE_1'-m^2-n\omega[E_1'-\cos\phi(E_1'^2-m^2)^{1/2}-E+\cos\psi(E^2-m^2)^{1/2}]\right.$$
$$\left.+\tfrac{1}{2}\nu^2\frac{[E_1'-\cos\phi(E_1'^2-m^2)^{1/2}-E+\cos\psi(E^2-m^2)^{1/2}]^2}{[E_1'-\cos\phi(E_1'^2-m^2)^{1/2}](E-\cos\psi(E^2-m^2)^{1/2})}\right\}.$$

If for a certain parameter set $|\cos\theta|\leqslant 1$ and the angle condition $|\psi-\theta|\leqslant\theta\leqslant\min(\psi+\phi, 2\pi-\psi-\phi)$ are fulfilled at the same time, the Møller cross section will be resonant.

For a qualitative examination the crude approximation $(EE_1'-m^2)/[(E^2-m^2)(E_1'^2-m^2)]^{1/2}\approx 1$ may be taken for the nonrelativistic energy region. Then the structure of the equation for cos θ becomes clearer:

$$\cos\theta\approx 1-n\omega f(E, E_1', \psi, \phi)+\nu^2 g(E, E_1', \psi, \phi).$$

Without an external field there is only the Coulomb divergence for $\theta = 0°$. The external field produces a set of resonances labelled by n. Since the function g is always positive, this n-multiplet is shifted into the unphysical region.

Explicit examples for resonance points are presented below in context with the results for the renormalised cross section. Besides the height, the width and spacing of the resonances will also be calculated.

5.1. *Renormalisation of the cross section*

The following radiative corrections must be inserted:

(*a*) Vacuum polarisation: The free photon propagator is replaced by (an approximation of) the full photon propagator in the external field. In this way the photon instability due to pair creation in the laser field is taken into account (Becker and Mitter 1975).

(*b*) Self-energy correction: The inclusion of the electron self-energy leads to a complex effective mass (Becker and Mitter 1976). The imaginary part of this effective mass corresponds to a finite line width of the quasi levels and hence to a finite life time (cf also Oleinik 1968).

A vertex correction is not important in this context since it does not affect the pole structure of the cross section.

For the damping of the resonant cross section by the vacuum polarisation the imaginary part of the photon mass is decisive. As will be shown below, a resonance behaviour can only be expected for non-relativistic electrons. For slow electrons and optical lasers, however, the imaginary part of the photon mass is very difficult to evaluate (Becker and Mitter 1975). Therefore the calculations of Ritus (1970) for a constant crossed field—the limiting case of the laser field—were used for an estimation of the magnitude of the correction term. The correction turned out to be extremely small so that the dominant part of the renormalisation must be the self-energy of the electrons.

For circular polarisation of the laser field the self-energy corrections were calculated by Becker and Mitter (1976): The magnitude of these corrections depends on the parameter

$$\rho = 2\frac{|p \, . \, k|}{m^2}.$$

For non-relativistic electrons and optical laser frequencies ρ is very small (between 10^{-7} and 10^{-5}). Then the pole of the corrected electron propagator is approximately determined by

$$p_\mu p^\mu \approx m^2(1 + \Delta m/m). \tag{5.1}$$

The real part of Δm can be neglected. The imaginary part is given by the following approximate formulae which are correct to a few per cent:

$$Jm(\Delta m/m) = \begin{cases} -0\,.\,145\alpha\rho(\nu^2)^{0\cdot 93} & 0\cdot 1 \lesssim \nu^2 \lesssim 1 \\ -\frac{1}{6}\alpha\rho\nu^2 & \nu^2 \lesssim 0\cdot 1. \end{cases} \tag{5.2}$$

Because of the complex mass of the electrons, their energy also obtains an imaginary part. Hence the energy of the incoming electrons must be replaced by $E - im\Gamma$ in the resonance denominator (Γ is twice the width of the quasi-levels). Neglecting terms proportional to Γ^2, we obtain from the imaginary part of the mass shell relation (5.1)

$$\Gamma = -(m/E)J_m(\Delta m(\rho)/m).$$

For the scattered electrons E' must be replaced by $E + \mathrm{i}m\Gamma'$ with

$$\Gamma' = -(m/E')J_m(\Delta m(\rho')/m).$$

These substitutions lead to an imaginary part R of the resonance denominator:

$$m^{-2}\tilde{\nu}(\tilde{p}_1' - \tilde{p}_1 + nk)^2 \rightarrow m^{-2}\tilde{\nu}(\tilde{p}_1' - \tilde{p}_1 + nk)^2 + \mathrm{i}R(n)$$

with

$$R(n) = (2/m)(E'\Gamma - E\Gamma') + 2n\frac{\omega}{m}(\Gamma + \Gamma') - \nu^2 \frac{E' - |\boldsymbol{p}'|\cos\phi - E + |\boldsymbol{p}|\cos\psi}{(E' - |\boldsymbol{p}'|\cos\phi)(E - |\boldsymbol{p}|\cos\psi)}$$
$$\times\Bigg[2(\Gamma + \Gamma') + (E' - |\boldsymbol{p}'|\cos\phi - E + |\boldsymbol{p}|\cos\psi)$$
$$\times\left(\frac{\Gamma}{E - |\boldsymbol{p}|\cos\psi} - \frac{\Gamma'}{E' - |\boldsymbol{p}'|\cos\phi}\right)\Bigg]$$

Γ and R are roughly proportional to ν^2. For a Nd–glass laser ($\omega/m = 2{\cdot}4 \times 10^{-6}$) with $\nu^2 = 1$ and for electrons at $5{\cdot}1$ eV, $r = 0$, $\psi = 45°$, $\phi = 44{\cdot}83°$ one obtains a radiative width Γ of $2{\cdot}5 \times 10^{-3}$ eV and $R = -2{\cdot}0 \times 10^{-13}$ (for the same parameters the imaginary part of the photon mass in the crossed-field approximation is $\exp(-10^{12})$).

Because of the renormalised resonance denominator the sum $S_r(b)$ must be redefined:

$$m^{-2}(\tilde{p}_1 - \tilde{p}_1 + nk)^2 + \mathrm{i}R(n) = 2\frac{p_1 \,.\, k - p_1' \,.\, k}{m^2}\left\{b - n + \mathrm{i}\frac{R(n)}{2(p_1 \,.\, k - p_1' \,.\, k)/m^2}\right\}$$

hence

$$\tilde{S}_r(b) = \sum_{n=-\infty}^{+\infty} \frac{J_n(\lambda_1)J_{-r-n}(\lambda_2)\exp(-\mathrm{i}nx)}{b - n + \mathrm{i}R(n)/[2(p_1 \,.\, k - p_1' \,.\, k)/m^2]}$$

must be inserted into the cross section at resonance (for all non-resonant cross sections $R(n)$ can be neglected even for high laser intensities).

At a resonance point $(b - n)$ in the denominator of $\tilde{S}_r(b)$ vanishes, i.e. b is an integer n^*. Therefore the height of the resonance depends on the magnitude of

$$\frac{J_{n^*}(\lambda_1)J_{-r-n^*}(\lambda_2)}{R(n^*)/(p_1 \,.\, k - p_1' \,.\, k)/m^2}.$$

A closer analysis of this expression shows that the following conditions are most favourable for a change in magnitude of the resonant cross section relative to cross sections in the vicinity of the resonance point:

—small (non-relativistic) electron energies

—high laser intensity

—high laser frequency

—small scattering angle.

For a Nd-glass laser, $\nu^2 = 1$ and $E_{\mathrm{kin}} = 5{\cdot}1$ eV, the angles $\psi \approx \phi \approx 45°$ and $\theta = |\psi - \phi|$ turn out to be the optimal combination. A smaller electron energy would produce higher resonances. But it is already questionable whether scattering experiments can be carried out with 5 eV electron beams.

Larger resonance effects can also be obtained by means of higher laser frequency, e.g. in the ultraviolet region. Although high intensities may not be reached with ultraviolet lasers in the near future, results will also be presented for this frequency region.

The following parameter sets were chosen for the numerical calculations of resonance points and the corresponding cross sections:

(i) $\omega = 1{\cdot}9\times10^{15}\ s^{-1}$ $\quad E_{\text{kin}} = 5{\cdot}1$ eV $\quad \psi = 45°$ $\quad \phi = 44{\cdot}83°$
$\theta = 0{\cdot}17°$

(ii) $\omega = 8{\cdot}0\times10^{15}\ \mathrm{s}^{-1}$ $\quad E_{\text{kin}} = 25{\cdot}5$ eV $\quad \psi = 45°$ $\quad \phi = 44{\cdot}83°$
$\theta = 0{\cdot}17°$.

For these parameter sets the intensity dependence of the resonant cross section is shown in figures 4 and 5. The resonance points were calculated for $r = 0$; there are no additional resonances for the same ν^2 for $r \neq 0$, since b varies considerably with r.

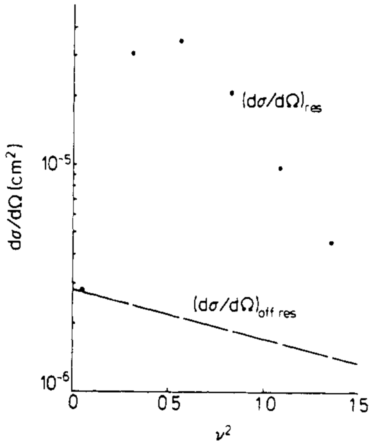

Figure 4. ν^2 dependence of the resonance cross section (in units cm^2) for $\omega = 8\times10^{15}\ \mathrm{s}^{-1}$, $E_{\text{kin}} = 25{\cdot}5$ eV, $\psi = 45°$, $\phi = 44{\cdot}83$, $\theta = 0{\cdot}17°$.

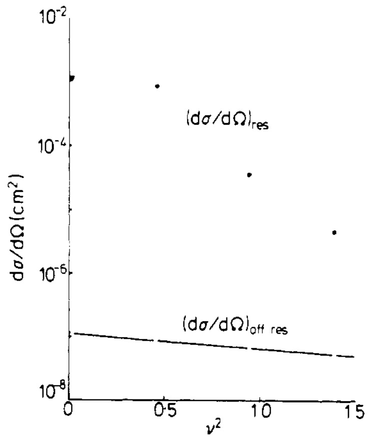

Figure 5. Cross sections at resonance (circles), off-resonance (broken curve) and free cross section (full curve) in units cm^2 as a function of the scattering angle θ for $\omega = 1{\cdot}9\times10^{15}\ \mathrm{s}^{-1}$, $\nu^2 = 0{\cdot}5$, $E_{\text{kin}} = 5{\cdot}1$ eV, $\psi = 45°$, $\phi = 44{\cdot}83°$.

No resonances were found for $\nu^2 < 10^{-2}$. There are further resonances for $\nu^2 > 1$, but formula (5.2), on which the calculations of the renormalised cross section are based, is only valid for $\nu^2 \lesssim 1$.

The deviations from the cross sections between two resonances are quite large for the ultraviolet laser; a factor 10^4 for $\nu^2 \approx 10^{-2}$. For the Nd-glass laser the maximum of the resonant cross section is ten times as large as the cross sections near the resonance point.

Because of the larger number of resonance points, the dependence of the resonances on ν^2 is better to be seen for the infrared laser: For a certain ν^2 there is a maximum value. For smaller and larger ν^2 the resonance elevation tends to zero. (A similar result is obtained for the ultraviolet laser, too, if the input parameters are varied slightly.) This somewhat surprising form of the ν^2 dependence is a consequence of expression (5.3). $R(n^*)$ is proportional to ν^2, hence the resonances should be higher for small ν^2. This tendency is counterbalanced by the Bessel functions which fall off very rapidly for small ν^2. On the other hand $R(n^*)$ becomes larger for $\nu^2 > 10^{-1}$ whereas the Bessel functions change only slightly with increasing ν^2 so that again the resonance will be small.

Calculations have also been done for higher electron energies: in the case of the Nd–glass laser (and the same angles as above) for 25 eV, 0·5 keV and 2·5 keV electrons, for the ultraviolet laser for 0·5 keV and 2·5 keV. In each case the resonance elevation dissappeared completely.

Therefore the somewhat unpleasant alternative for experiments is either to take an infrared laser—for which high intensities are attainable—and to put up with very small electron energies, or to use faster electrons and a laser with higher frequency. Finally, the dependence of the resonances on the scattering angle θ is presented in figure 6 for the first parameter set in order to confirm for this example that the resonance indeed reaches its maximum for the minimum value of θ. Clearly the height of the resonances decreases rapidly for increasing θ (ψ and ϕ fixed). For smaller differences $|\psi - \phi|$ the resonances become higher if the minimal θ is chosen. Thus an experiment should be carried out at the smallest still measurable θ.

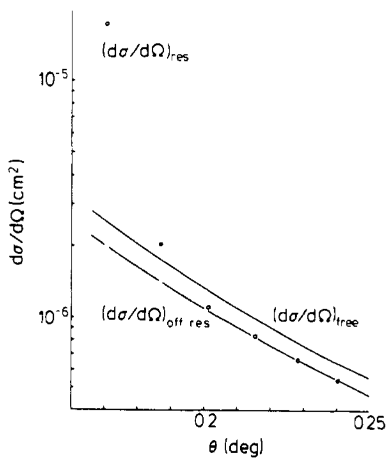

Figure 6. Resonance widths $2\Delta\theta$ (10^{-4} deg) and $2\Delta E(10^{-2}\,\text{eV})$ for $\omega = 1{\cdot}9 \times 10^{15}\,\text{s}^{-1}$, $E_{\text{kin}} = 5{\cdot}1$ eV, $\psi = 45°$, $\phi = 44{\cdot}83°$, $\theta = 0{\cdot}17°$.

5.2. *Width of the resonances*

Because of the complicated form of the cross section one cannot assume a Breit–Wigner form for the resonance width, so one has to start from the definition. The resonance width Δx for a parameter x is chosen to satisfy the condition that the cross section at the points $x = x_R + \Delta x$ is half the resonant value at x_R. The cross section has the following structure:

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}\Omega} \sim \left| \sum_{n=-\infty}^{+\infty} \left(\frac{A}{(\tilde{p}_1' - \tilde{p}_1 + nk)^2 + \mathrm{i}R(n)} - \text{exchange term} \right) \right|^2 .$$

In the case of a high resonance the exchange term can be neglected since in general the denominator will not vanish at the same parameter values as the denominator of the first term. Further approximations finally yield a formula for the width of a high resonance:

$$\Delta x = \left| \frac{R(x_R, n^*)}{(\mathrm{d}/\mathrm{d}x)(\tilde{p}_1' - \tilde{p}_1 + n^* k)^2 |_{x=x_R}} \right| . \tag{5.3}$$

Since R is proportional to ν^2, the resonances will be very narrow for small ν^2. The interesting widths are those for the energy and the angles. In the case of the scattering angle θ the calculation is easy since R does not depend on θ.

$$\Delta\theta \approx \left(2 \frac{|\boldsymbol{p}|}{m} \frac{|\boldsymbol{p}_1'|}{m} \right)^{-1} \left| \frac{R(n^*)}{\sin \theta_R} \right| .$$

Thus $\Delta\theta$ is inversely proportional to the electron momenta and $\sin \theta_R$ and—because of R—directly proportional to ν^2. Hence the resolving power relative to θ will be good for high laser intensities, small electron momenta and small scattering angles—preconditions which are also needed for high resonances.

The calculation of ΔE is more difficult. Therefore the numerical results are directly given in figures 7 and 8 for the same parameter sets as in figures 4 and 5. In all cases the widths are very small (no line broadening effects have been considered, e.g. the fact that a laser is really not strictly monochromatic).

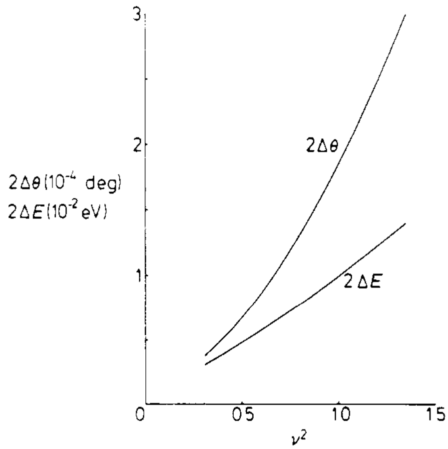

Figure 7. Resonance widths $2\Delta\theta$ (10^{-4} deg) and $2\Delta E$ (10^{-2} eV) for $\omega = 8{\cdot}0 \times 10^{15}\ \mathrm{s}^{-1}$, $E_{\mathrm{kin}} = 25{\cdot}5$ eV, $\psi = 45°$, $\phi = 44{\cdot}83$, $\theta = 0{\cdot}17°$.

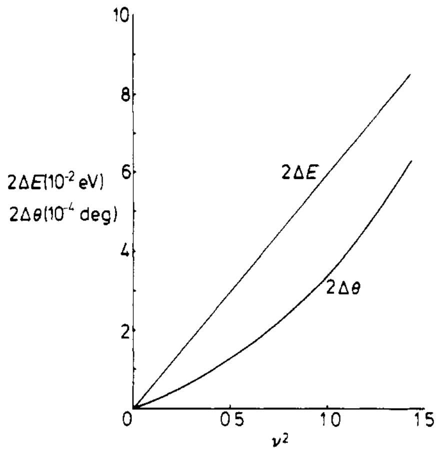

Figure 8. Resonance width for $\omega = 8\times10^{15}\ s^{-1}$, $E_{\mathrm{kin}} = 25\cdot5$ eV, $\psi = 45°$, $\phi = 44\cdot83°$, $\theta = 0\cdot17°$.

5.3. Spacing of the resonances

For the experimental detection of resonances the question is important whether the resonances overlap. An explicit approximation formula for the distance between the resonances can be given for θ. For the other parameters the spacing must be calculated numerically. The distance between two resonances on the θ scale, $\delta\theta$, is obtained from a nonrelativistic expansion of the resonance condition for small angles θ and $\delta\theta \ll \theta$:

$$\delta\theta \approx \frac{\omega}{\theta}\,\frac{2|\boldsymbol{p}|\cos\psi - 2|\boldsymbol{p}_1'|\cos\phi + |\boldsymbol{p}_1'|^2 - |\boldsymbol{p}|^2}{2|\boldsymbol{p}'||\boldsymbol{p}_1'|}.$$

Again a high laser frequency, small electron energies and scattering angles are favourable.

In the following tables resonance spacing and width are compared in two examples for each parameter set (tables 1 and 2):

Table 1. $\omega = 1\cdot9\times10^{15}\ \mathrm{s}^{-1}$ $E_{\mathrm{kin}} = 5\cdot1$ eV $\psi = 45°$ $\phi = 44\cdot83°$ $\theta = 0\cdot17°$

ν^2	$\delta\theta$ (deg)	$2\Delta\theta$ (deg)	δE (eV)	$2\Delta E$ (eV)
0·31	$1\cdot7\times10^{-2}$	$2\cdot8\times10^{-5}$	2·0	0·003
1·09	$1\cdot1\times10^{-2}$	$2\cdot2\times10^{-4}$	1·1	0·011

Table 2. $\omega = 8\cdot0\times10^{15}\ s^{-1}$ $E_{\mathrm{kin}} = 25\cdot5$ eV $\psi = 45°$ $\phi = 44\cdot83°$ $\theta = 0\cdot17°$

ν^2	$\delta\theta$ (deg)	$2\Delta\theta$ (deg)	δE (eV)	$2\Delta E$ (eV)
0·01	$3\cdot4\times10^{-2}$	$1\cdot9\times10^{-6}$	25·0	0·001
0·94	$2\cdot6\times10^{-2}$	$3\cdot1\times10^{-4}$	12·5	0·03

The values of the spacing relative to ψ and ϕ are aproximately the same as for θ. The examples show clearly that the distances between the resonances are much larger than their width.

6. Conclusions

The results of § 4 and 5 show clearly that the effects produced by a strong laser in Møller scattering are large enough for experimental verification. A necessary precondition, however, is a very high laser intensity.

For the measurement of the intensity-dependent shift of the cross-section in infrared laser (e.g. Nd laser) is sufficient for which $\nu^2 \approx 1$ seems to be attainable. For certain electron parameters the change in cross-section relative to free Møller scattering is considerable—more than 100% for $\nu^2 = 1$. Furthermore the effect appears not to be restricted to very slow electrons.

In the case of the resonances the effects are, of course, much larger. Apart from the required strong laser field, the experimental requirements are different. If an infrared laser is used, high resonances can be expected only for very slow electrons ($\approx$5 eV). For an ultraviolet laser slightly higher electron energies ($\approx$25 eV) may be taken, but then one has the problem of reaching high intensities ($\nu^2 \gtrsim 10^{-2}$) with such a laser. In any case a high angular resolving power is required since the scattering angle must be as small as possible and the angular spacing of the resonances is rather narrow. Thus the experimental proof of the existence of resonances will be difficult, but not impossible. It will depend on the technical perfection of the apparatus used.

Acknowledgments

We want to thank Dr W Becker for numerous helpful discussions and valuable suggestions.

Appendix A. Light-like coordinates

From the orthogonal set of unit vectors $\boldsymbol{n}$ and $\boldsymbol{e}_\iota$ the four-vectors

$$n^\mu = 2^{-1/2}(1, \boldsymbol{n}), \qquad \hat{n}^\mu = 2^{-1/2}(1, -\boldsymbol{n}), \qquad e_\iota^\mu = (0, \boldsymbol{e}_\iota)$$

can be constructed which serve as a base of the Minkowski space. The vectors n and $\hat{n}$ are light-like, i.e. $n^2 = \hat{n}^2 = 0$, $n \,.\, \hat{n} = 1$. Instead of e_1 and e_2 the vectors

$$e_\pm^\mu = (e_1^\mu \pm \mathrm{i} e_2^\mu)$$

with $e_+e_- = -1$, $e_+^2 = e_-^2 = e_\pm \,.\, n = e_\pm \,.\, \hat{n} = 0$ may be introduced.

Any vector p can be decomposed according to

$$p^\mu = n^\mu p_u + \hat{n}^\mu p_v + e_\iota^\mu p_\iota = n^\mu p_u + \hat{n}^\mu p_v + e_+ p_- + e_- p_+$$

with the components

$$p_u = p \,.\, \hat{n} \qquad p_v = p \,.\, n \qquad p_\iota = -p \,.\, e_\iota \qquad p_\pm = -p \,.\, e_\pm .$$

A special notation is used for the coordinate vector:

$$v = x_u, \qquad u = x_v.$$

With this notation the scalar product $p \, . \, x$ has the form

$$p \, . \, x = p_u u + p_v v - p_t x_t = p_u u + p_v v - p_+ x_- - p_- x_+.$$

The algebra for the light-like components γ_u, γ_v, $\gamma_\pm$ of the Dirac matrices is particularly advantageous for many problems. Detailed information may be found in Mitter (1975).

Appendix B. Explicit form of electron momenta and differential cross section (for a numerical treatment)

According to the reference system defined in § 3 the momenta of the incoming electrons are given by $E_1 = E_2 = E$, $\boldsymbol{p}_1 = -\boldsymbol{p}_2 = \boldsymbol{p}$. The polarisation vectors $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ and the propagation vector $\boldsymbol{n}$ are chosen as a base for the three-dimensional space. Using the cylindrical symmetry of the laser field the incoming electrons may be chosen to be in the $\boldsymbol{e}_1$–$\boldsymbol{e}_2$ plane and we have

$$p_1 = (E, 0, (E^2 - m^2)^{1/2} \sin \psi, (E^2 - m^2)^{1/2} \cos \psi)$$

$$p_2 = (E, 0, -(E^2 - m^2)^{1/2} \sin \psi, -(E^2 - m^2)^{1/2} \cos \psi).$$

E_1' must be calculated numerically from (3.2) (Brock 1977).

For $\sin \psi \neq 0$ one obtains

$$p_1' = (E_1', [(E_1'^2 - m^2)(\sin^2 \phi - (\cos \theta - \cos \psi \cos \phi)^2/\sin^2 \psi)]^{1/2},$$
$$\times (E_1'^2 - m^2)^{1/2}(\cos \theta - \cos \psi \cos \phi/\sin \psi), (E_1'^2 - m^2)^{1/2} \cos \phi).$$

If the electrons come in parallel or antiparallel to the laser, i.e. $\sin \psi = 0$, there is an additional degree of freedom because of the cylindrical symmetry of the system of laser plus incoming electrons. If we chose $\boldsymbol{p}_1' \, . \, \boldsymbol{e}_1 = 0$ we obtain

$$p_1' = (E_1', 0, (E_1'^2 - m^2)^{1/2} \sin \phi, (E_1'^2 - m^2)^{1/2} \cos \phi).$$

The components of p_2 are given by

$$\rho_2' = \left(\frac{E_1'(E_1' - (E_1'^2 - m^2)^{1/2} \cos \phi) + 2E(E - E_1' + (E_1'^2 - m^2)^{1/2} \cos \phi)}{2(2E - E_1' + (E_1'^2 \propto)^{1/2} \cos \phi)}, \right.$$
$$\left. -\boldsymbol{p}_1' \, . \, \boldsymbol{e}_1, -\boldsymbol{p}_1 \, . \, \boldsymbol{e}_2, E_2' - 2E + E_1' - (E_1'^2 - m^2)^{1/2} \cos \phi \right).$$

The differential cross section is given by (3.3). The parts of M_r containing Dirac matrices have the form

$$U(\Gamma_1, \Gamma_2) = [\bar{u}(p_1', s_1')\Gamma_1^\mu(p_1', p_1)u(p_1, s_1)][\bar{u}(p_2', s_2')\Gamma_{2\mu}(p_2', p_2)u(\rho_2, s_2)],$$

where Γ_i^μ stands for B^μ, $C_\pm^\mu$. With the infinite sums $S_r(b)$ defined in (3.4) and

$\rho := \rho(p_2', p_2)$, M_r is given by

$$M_r = \frac{m^4}{16} \sum_{s_1, s_2, s_1', s_2'} \left| \frac{\exp(-\mathrm{i}r\rho)}{p_1 \cdot k - p_1' \cdot k} \left\{ S_r(b) U(B, B) - S_{r+1}(b) \exp(-\mathrm{i}\rho) U(B, C_+) \right. \right.$$
$$- S_{r-1}(b) \exp(\mathrm{i}\rho) U(B, C_-) - S_{r+1}(b-1) \exp(-\mathrm{i}\rho) U(C_+, B)$$
$$+ S_{r+2}(b-1) \exp(-2\mathrm{i}\rho) U(C_+, C_+) + S_r(b-1) U(C_+, C_-)$$
$$- S_{r-1}(b+1) \exp(\mathrm{i}\rho) U(C_-, B) + S_r(b+1) \exp(\mathrm{i}\rho) U(C_- C_+)$$
$$\left. \left. + S_{r-2}(b+1) \exp(2\mathrm{i}\rho) U(C_-, C_-) \right\} - (p_1', s_1') \leftrightarrow (p_2', s_2') \right|^2.$$

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